30 Higher and Super Definite Integrals of Linear Composite Functions

Abstract

In this chapter, we study definite integrals, higher-order definite integrals and super definite integrals for a composite function $g(f(x))$ where $f(x)$ is a linear function.

(1) In Section 1, we derive the theorem for the definite integral of $g(cx+d)$ ..

(2) In Section 2, we progress (1) to higher definite integrals.

(3) In Section 3, we progress (2) to super definite integrals.

30.1 Definite Integrals of Linear Composite Functions

Theorem 30.1.1

Let c be a positive number and a, b, d are real numbers. If a primitive function $g^{<1>}(x)$ of a function $g(x)$ is bounded and holomorphic at an interval $[ac+d, bc+d]$, the following holds.

$$\int_a^b g(cx+d)\,dx = \frac{1}{c}\int_{ac+d}^{bc+d} g(x)\,dx$$

Proof

Let $cx+d = f$, $ac+d = f_a$. Then $dx = df/c$.

Therefore, if we perform the variable transformation $[a, x] \rightarrow [f_a, f]$,

$$\int_a^x g(cx+d)\,dx = \int_{f_a}^{f} g(f)\frac{df}{c}$$

Here, put $x=b$, then $bc+d = f_b$. Therefore,

$$\int_a^b g(cx+d)\,dx = \frac{1}{c}\int_{f_a}^{f_b} g(f)\,df$$

i.e.

$$\int_a^b g(cx+d)\,dx = \frac{1}{c}\int_{ac+d}^{bc+d} g(f)\,df$$

In the integral on the right hand side, both the upper and lower limits are constants, so the symbol f can be changed freely. Thus, changing f to x , we obtain the desired expression. Q.E.D.

Example 1 $g(cx+d) = (cx+d)^3 - 3(cx+d) + 1$

According to the theorem, the definite integral from 2 to 7 is as follows

$$\int_2^7 \left\{(cx+d)^3 - 3(cx+d) + 1\right\}dx = \frac{1}{c}\int_{2c+d}^{7c+d}\left(x^3 - 3x + 1\right)dx$$

Calculating both sides by the mathematical processing software *Mathematica*,

```
Gl[c, d] :          5 - 135 c/2 + 2385 c^3/4 - 15 d + 335 c^2 d + 135 c d^2/2 + 5 d^3
Expand[Gr[c, d]] :  5 - 135 c/2 + 2385 c^3/4 - 15 d + 335 c^2 d + 135 c d^2/2 + 5 d^3
```

$$Gl[c,d]: \quad 5 - \frac{135\,c}{2} + \frac{2385\,c^3}{4} - 15\,d + 335\,c^2\,d + \frac{135\,c\,d^2}{2} + 5\,d^3$$

$$Expand[Gr[c,d]]: \quad 5 - \frac{135\,c}{2} + \frac{2385\,c^3}{4} - 15\,d + 335\,c^2\,d + \frac{135\,c\,d^2}{2} + 5\,d^3$$

It turns out that both sides are equal.

Example 2 $g(cx+d) = (cx+d)^2 cos(cx+d)$

According to the theorem, the definite integral from 1 to 5 is as follows

$$\int_1^5 (cx+d)^2 cos(cx+d)\,dx = \frac{1}{c}\int_{1c+d}^{5c+d} x^2 cos\,x\,dx$$

When $c=3, d=4$, calculating both sides by *Mathematica* ,

Gl[3, 4] : $\frac{1}{3}$ (-14 Cos[7] + 38 Cos[19] - 47 Sin[7] + 359 Sin[19])

Gr[3, 4] : $\frac{1}{3}$ (-14 Cos[7] + 38 Cos[19] - 47 Sin[7] + 359 Sin[19])

It turns out that both sides are equal.

Setting $a=\pm\infty$, $b=0$ in Theorem 30.1.1 , we obtain the following corollary.

Corollary 30.1.2

Let c be a positive numbe, d be a real number and n be a nartural number, If a primitive function $g^{<1>}(x)$ of a function $g(x)$ is bounded and holomorphic at an interval $(\pm\infty, 0]$, the following holds.

$$\int_{\pm\infty}^{0} g(cx+d)\,dx = \frac{1}{c}\int_{\pm\infty}^{d} g(x)\,dx$$

Where, for $\pm$, the one where the integrals converge is used.

The difficulty with this corollary is that " the primitive function $g^{<1>}(x)$ is bounded in the interval $(\pm\infty, 0]$ " What kind of primitive function satisfies this condition? Through trial and error, I have found that the primitive functions satisfy this condition approximately in the following cases.

Cases where the primitive function satisfies the conditions of the theorem.

(1) $g^{<1>}(x)$ has $arctan\,x$, $arccot\,x$ as a factor.

(2) $g^{<1>}(x)$ has $tanh\,x$ as a factor.

(3) $g^{<1>}(x)$ has a fractional function with equal degrees of numerator and denominator as a factor.

(4) $g^{<1>}(x)$ has e^x as a factor.

Example 1 $g^{<1>}(x)$ **has** $arctan\,x$ **as a factor**

$$\int_{\pm\infty}^{0} \frac{dx}{(cx+d)^2+1} = \frac{1}{c}\int_{\pm\infty}^{d} \frac{dx}{x^2+1}$$

Left:

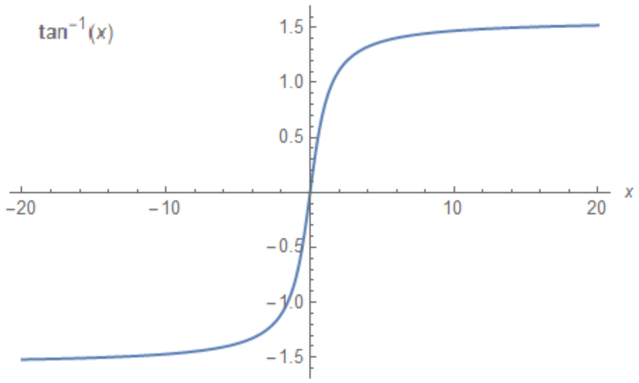
$$\int_{\pm\infty}^{0} \frac{dx}{(cx+d)^2+1} = \left[\int \frac{dx}{(cx+d)^2+1}\right]_{\pm\infty}^{0} = \left[\frac{tan^{-1}(cx+d)}{c}\right]_{\pm\infty}^{0}$$

$$= \frac{1}{c}\left\{\tan^{-1}d - \tan^{-1}\infty\right\} = \frac{1}{c}\left(\tan^{-1}d \mp \frac{\pi}{2}\right)$$

Right:

$$\frac{1}{c}\int_{\pm\infty}^{d}\frac{dx}{x^2+1} = \left[\int\frac{dx}{x^2+1}\right]_{\pm\infty}^{d} = \frac{1}{c}\left[\tan^{-1}x\right]_{\infty}^{d} = = \frac{1}{c}\left(\tan^{-1}d \mp \frac{\pi}{2}\right)$$

These are the consequences of being $\lim_{x\to\pm\infty}\tan^{-1}(cx+d) = \lim_{x\to\pm\infty}\tan^{-1}x = \pm\pi/2$. (See figure.)



In additiona, a similar figure can also be drawn for (2) and (3) above.

Example 2 $g^{<1>}(x)$ **has** e^x **as a factor**

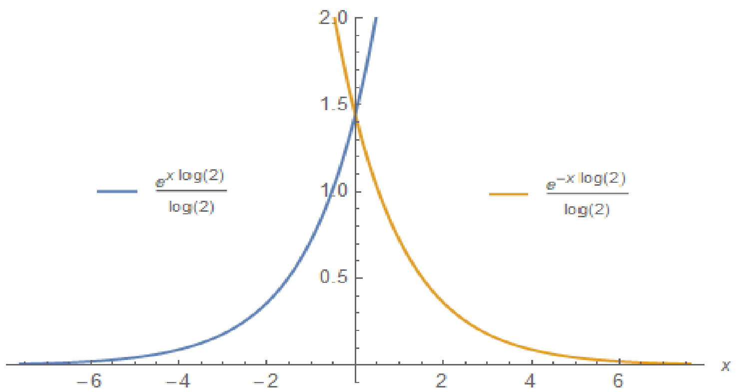
$$\int_{-\infty}^{0}\lambda^{cx+d}\,dx = \frac{1}{c}\int_{-\infty}^{d}\lambda^{x}\,dx \qquad (\lambda > 0)$$

Left: $$\int_{-\infty}^{0}\lambda^{cx+d}\,dx = \left[\frac{1}{c}\frac{e^{(cx+d)\log\lambda}}{\log\lambda}\right]_{-\infty}^{0} = \frac{e^{d\log\lambda}}{c\log\lambda}$$

Right: $$\frac{1}{c}\int_{-\infty}^{d}\lambda^{x}\,dx = \frac{1}{c}\left[\frac{e^{x\log\lambda}}{\log\lambda}\right]_{-\infty}^{d} = \frac{e^{x\log\lambda}}{c\log\lambda}$$

These are the consequences of being $\lim_{x\to-\infty}e^{(cx+d)\log\lambda} = \lim_{x\to-\infty}e^{x\log\lambda} = 0$. (See figure.)

When $+$, same as above.



The power of $e^{\mp\infty}$ in Examplw 2 is enormous. Therefore, most $g^{<1>}(x)$ including exponential functions are considered to satisfy the conditions of Corollary 30.1.2 . In fact, here is a list of what has been verified.

$$\int_{\mp\infty}^{0} e^{\pm(cx+d)}\,dx = \frac{1}{c}\int_{\mp\infty}^{d} e^{\pm x}\,dx$$

$$\int_{\mp\infty}^{0} \lambda^{\pm(cx+d)}\,dx = \frac{1}{c}\int_{\mp\infty}^{d} \lambda^{\pm x}\,dx \qquad (\lambda > 1)$$

$$\int_{\pm\infty}^{0} e^{-\lambda(cx+d)^2}dx = \frac{1}{c}\int_{\pm\infty}^{d} e^{-\lambda x^2}dx$$

(Gaussian Integral half)

$$\int_{\infty}^{-\infty} e^{-\lambda(cx+d)^2}dx = \frac{1}{c}\int_{\infty}^{-\infty} e^{-\lambda x^2}dx$$

(Gaussian Integral)

$$\int_{-\infty}^{0} Ei(cx+d)\,dx = \frac{1}{c}\int_{-\infty}^{d} Ei(x)\,dx \qquad \left(Ei(x) = \int_{-\infty}^{x} \frac{e^x}{x}dx \right)$$

$$\int_{\mp\infty}^{0} e^{\pm(cx+d)}(cx+d)^{\lambda}\,dx = \frac{1}{c}\int_{\mp\infty}^{d} e^{\pm x}\,x^{\lambda}\,dx \qquad (\lambda > -1)$$

$$\int_{\mp\infty}^{0} e^{\pm(cx+d)}\,\log(cx+d)\,dx = \frac{1}{c}\int_{\mp\infty}^{d} e^{\pm x}\,\log x\,dx$$

$$\int_{\mp\infty}^{0} e^{\pm(cx+d)}\,\sin(cx+d)\,dx = \frac{1}{c}\int_{\mp\infty}^{d} e^{\pm x}\,\sin x\,dx$$

$$\int_{\mp\infty}^{0} e^{\pm(cx+d)}\,\cos(cx+d)\,dx = \frac{1}{c}\int_{\mp\infty}^{d} e^{\pm x}\,\cos x\,dx$$

$$\vdots$$

30.2 Higher Definite Integrals of Linear Composite Functions

Lemma 30.2.0

When $cx+d=f,\ ca+d=f_a$, the following expression holds for a natural number n .

$$\int_a^x\cdots\int_a^x g(cx+d)dx^n = \left(\frac{1}{c}\right)^n \int_{f_a}^f\cdots\int_{f_a}^f g(f)df^n$$

Proof

By assumption, $dx=df/c$. So, if we perform the variable transformation $[a,x]\rightarrow[f_a,f]$,

$$\int_a^x g(cx+d)dx = \frac{1}{c}\int_{f_a}^f g(f)df$$

Integrating both sides from a to x

$$\int_a^x\int_a^x g(cx+d)dx^2 = \int_a^x\left(\frac{1}{c}\int_{f_a}^f g(f)df\right)dx$$

Substituting $dx=df/c$ for the right side and perform the transformation $[a,x]\rightarrow[f_a,f]$,

$$\int_a^x\int_a^x g(cx+d)dx^2 = \int_{f_a}^f\left(\frac{1}{c}\int_{f_a}^f g(f)df\right)\frac{df}{c}$$

i.e.

$$\int_a^x\int_a^x g(cx+d)dx^2 = \left(\frac{1}{c}\right)^2\int_{f_a}^f\int_{f_a}^f g(f)df^2$$

Hereafter, repeating the same calculations, we obtain the desired expression. Q.E.D.

Note

This lemma is the same as Formual 23.1.2 (" 23 Higher Integral of Composition ") . The formula was proved after a lengthy calculation from a general formula for higher integrals of composite functions, but the lemma in this section is proved directly.

Using this lemma, we can prove the following theorem.

Theorem 30.2.1

Let $c>0$, a,b,d are real numbers and n be a natural number, If a higher primitive function $g^{<n>}(x)$ of a function $g(x)$ is bounded and holomorphic at an interval $[ac+d,bc+d]$, the following holds.

$$\int_a^b\int_a^x\cdots\int_a^x g(cx+d)dx^n = \left(\frac{1}{c}\right)^n\int_{ac+d}^{bc+d}\int_{ac+d}^x\cdots\int_{ac+d}^x g(x)dx^n$$

Proof

According Lemma 30.2.0 , when $cx+d=f,\ ac+d=f_a$ the following holds for a natural number n ,

$$\int_a^x\cdots\int_a^x g(cx+d)dx^n = \left(\frac{1}{c}\right)^n\int_{f_a}^f\cdots\int_{f_a}^f g(f)df^n$$

Here, put $x=b$, then $bc+d=f_b$. Therefore,

$$\int_a^b \int_a^x \cdots \int_a^x g(cx+d)\,dx^n = \left(\frac{1}{c}\right)^n \int_{f_a}^{f_b} \int_{f_a}^{f} \cdots \int_{f_a}^{f} g(f)\,df^n$$

i.e.

$$\int_a^b \int_a^x \cdots \int_a^x g(cx+d)\,dx^n = \left(\frac{1}{c}\right)^n \int_{ac+d}^{bc+d} \int_{ac+d}^{f} \cdots \int_{ac+d}^{f} g(f)\,df^n$$

In the higher integral on the right hand side, both the upper and lower limits are constants, so the symbol f can be changed freely. Thus, changing f to x, we obtain the desired expression. Q.E.D.

Example 1 $g(cx+d) = \lambda^{cx+d}$

According to the theorem, the 3 rd order definite integral from 1 to 6 is as follows

$$\int_1^6 \int_1^x \int_1^x \lambda^{cx+d} dx^3 = \left(\frac{1}{c}\right)^n \int_{1c+d}^{6c+d} \int_{1c+d}^{x} \int_{1c+d}^{x} \lambda^x\,dx^3$$

Since this is a conceptual formula, the following triple integral is required for actual calculation.

$$\int_1^6 \int_1^u \int_1^t \lambda^{ct+d} dt\,du\,dx = \left(\frac{1}{c}\right)^n \int_{1c+d}^{6c+d} \int_{1c+d}^{u} \int_{1c+d}^{t} \lambda^t\,dt\,du\,dx$$

When $\lambda=2$, calculating both sides by *Mathematica*,

Expand[Gl[c, d, 2]] : $-\frac{2^{c+d}}{c^3\,\mathrm{Log}[2]^3} + \frac{2^{6c+d}}{c^3\,\mathrm{Log}[2]^3} - \frac{5\times 2^{c+d}}{c^2\,\mathrm{Log}[2]^2} - \frac{25\times 2^{-1+c+d}}{c\,\mathrm{Log}[2]}$

Expand[Gr[c, d, 2]] : $-\frac{2^{c+d}}{c^3\,\mathrm{Log}[2]^3} + \frac{2^{6c+d}}{c^3\,\mathrm{Log}[2]^3} - \frac{5\times 2^{c+d}}{c^2\,\mathrm{Log}[2]^2} - \frac{25\times 2^{-1+c+d}}{c\,\mathrm{Log}[2]}$

It turns out that both sides are equal.

Example 2 $g(cx+d) = (cx+d)^3 cos(cx+d)$

According to the theorem, the 3 rd order definite integral from 2 to 5 is as follows

$$\int_2^5 \int_2^u \int_2^t (ct+d)^3 cos(ct+d)\,dt\,du\,dx = \left(\frac{1}{c}\right)^n \int_{2c+d}^{5c+d} \int_{2c+d}^{u} \int_{2c+d}^{t} t^3 cos\,t\,dt\,du\,dx$$

When $c=3, d=4$, calculating both sides by *Mathematica*,

Gl[c_, d_] := $\int_2^5 \left(\int_2^x \left(\int_2^u (c\,t+d)^3\,\mathrm{Cos}[c\,t+d]\,dt\right) du\right) dx$

Gr[c_, d_] := $\left(\frac{1}{c}\right)^3 \int_{2c+d}^{5c+d} \left(\int_{2c+d}^{x} \left(\int_{2c+d}^{u} t^3\,\mathrm{Cos}[t]\,dt\right) du\right) dx$

```
N[{Gl[3, 4], Gr[3, 4]}]
{822.152    , 822.152}
```

It turns out that both sides are equal.

Calculations like the two Examples above can be done using multiple integrals up to the thi3 rd or 4 th order, but when it comes to the 5 th or 8 th order, it becomes difficult to describe and calculate.

To solve this problem, we will derive the following theorem using Cauchy Formula for Repeated Integration (" 4. Higher Integral ").

Theorem 30.2.1'

Let $c>0$, a,b,d are real numbers and n be a natural number, If a higher primitive function $g^{<n>}(x)$ of a function $g(x)$ is bounded and holomorphic at an interval $[ac+d, bc+d]$, the following holds.

$$\frac{1}{\Gamma(n)}\int_a^b (b-x)^{n-1} g(cx+d)dx = \left(\frac{1}{c}\right)^n \frac{1}{\Gamma(n)}\int_{ac+d}^{bc+d} (bc+d-x)^{n-1} g(x)dx$$

Proof

According to Theorem 4.2.1 (" 4. Higher Integral "), Cauchy Formula for Repeated Integration is as follows.

$$\int_a^x \cdots \int_a^x g(x)dx^n = \frac{1}{\Gamma(n)}\int_a^x (x-t)^{n-1} g(t)dt$$

Using this, Lemma 30.2.0 can be described as follows.

$$\frac{1}{\Gamma(n)}\int_a^x (x-t)^{n-1} g(ct+d)dt = \left(\frac{1}{c}\right)^n \frac{1}{\Gamma(n)}\int_{f_a}^{f} (f-t)^{n-1} g(f)df$$

Here, put $x=b$, then $bc+d = f_b$. Therefore,

$$\frac{1}{\Gamma(n)}\int_a^b (b-t)^{n-1} g(ct+d)dt = \left(\frac{1}{c}\right)^n \frac{1}{\Gamma(n)}\int_{f_a}^{f_b} (f_b-t)^{n-1} g(f)df$$

i.e.

$$\frac{1}{\Gamma(n)}\int_a^b (b-t)^{n-1} g(ct+d)dt = \left(\frac{1}{c}\right)^n \frac{1}{\Gamma(n)}\int_{ac+d}^{bc+d} (bc+d-t)^{n-1} g(f)df$$

In the higher integral on the right hand side, both the upper and lower limits are constants, so the symbol f can be changed freely. Thus, changing f to t, further, changing t to x in both sides, we obtain the desired expression. Q.E.D.

Example 1' $g(cx+d) = \lambda^{cx+d}$

According to the theorem, the n th order definite integral from 1 to 6 is as follows

$$\frac{1}{\Gamma(n)}\int_1^6 (6-x)^{n-1}\lambda^{cx+d}\, dt = \left(\frac{1}{c}\right)^n \frac{1}{\Gamma(n)}\int_{1c+d}^{6c+d} (6c+d-x)^{n-1}\lambda^x dx \quad (\lambda>1)$$

When $\lambda=2$, calculating both sides by *Mathematica* for $n=3$,

Expand[Gl[c, d, 2, 3]] : $-\frac{2^{c+d}}{c^3 \operatorname{Log}[2]^3}+\frac{2^{6c+d}}{c^3 \operatorname{Log}[2]^3}-\frac{5\times 2^{c+d}}{c^2 \operatorname{Log}[2]^2}-\frac{25\times 2^{-1+c+d}}{c \operatorname{Log}[2]}$

Expand[Gr[c, d, 2, 3]] : $-\frac{2^{c+d}}{c^3 \operatorname{Log}[2]^3}+\frac{2^{6c+d}}{c^3 \operatorname{Log}[2]^3}-\frac{5\times 2^{c+d}}{c^2 \operatorname{Log}[2]^2}-\frac{25\times 2^{-1+c+d}}{c \operatorname{Log}[2]}$

The results are in exact agreement with Example 1 .

Setting $a=\pm\infty$, $b=0$ in Theorem 30.2.1' , we obtain the following corollary.

Corollary 30.2.2'

Let c be a positive numbe, d be a real number and n be a nartural number, If a higher order primitive function $g^{<n>}(x)$ of a function $g(x)$ is bounded and holomorphic at an interval $(\pm\infty, 0]$, the following holds.

$$\frac{1}{\Gamma(n)}\int_{\pm\infty}^{0}(0-x)^{n-1}g(cx+d)dx = \left(\frac{1}{c}\right)^{n}\frac{1}{\Gamma(n)}\int_{\pm\infty}^{d}(d-x)^{n-1}g(x)dx$$

Where, for $\pm$, the one where the integrals converge is used.

Example 2' $g(cx+d) = log\left(tanh\frac{cx+d}{2}\right)$

According to the corollary., the n th order definite integral from ∞ to 0 is as follows

$$\frac{1}{\Gamma(n)}\int_{\infty}^{0}(-x)^{n-1}log\left(tanh\frac{cx+d}{2}\right)dx = \left(\frac{1}{c}\right)^{n}\frac{1}{\Gamma(n)}\int_{\infty}^{d}(d-x)^{n-1}log\left(tanh\frac{x}{2}\right)dx$$

When $c=3, d=4$, calculating both sides by *Mathematica* for $n=5$,

```
Γ[n_] := Gamma[n]
Gl[c_, d_, n_] := 1/Γ[n] ∫_∞^0 (0 - x)^(n-1) Log[Tanh[(c x + d)/2]] ⅆx
Gr[c_, d_, n_] := (1/c)^n 1/Γ[n] ∫_∞^d (d - x)^(n-1) Log[Tanh[x/2]] ⅆx
N[{Gl[3, 4, 5], Gr[3, 4, 5]}]
{0.000150746 , 0.000150746}
```

It turns out that both sides are equal.

30.3 Super Definite Integrals of Linear Composite Functions

Theorem 30.3.1

Let c, p are a positive numbers and a, b, d are real numbers, If a super primitive function $g^{<p>}(x)$ of a function $g(x)$ is bounded and holomorphic at an interval $[ac+d, bc+d]$, the following holds.

$$\frac{1}{\Gamma(p)}\int_a^b (b-x)^{p-1} g(cx+d)\,dx = \left(\frac{1}{c}\right)^p \frac{1}{\Gamma(p)}\int_{ac+d}^{bc+d} (bc+d-x)^{p-1} g(x)\,dx$$

Proof

Replacing the natural number n with a positive number p in Theorem 30.2.1' , we obtain the desired expression. Q.E.D.

Example 1 $g(cx+d) = sech(cx+d)$

According to the theorem, the p th order definite integral from 2 to 10 is as follows

$$\frac{1}{\Gamma(p)}\int_2^{10} (10-x)^{p-1} sech(cx+d)\,dx = \left(\frac{1}{c}\right)^p \frac{1}{\Gamma(p)}\int_{2c+d}^{10c+d} (10c+d-x)^{p-1} sech\,x\,dx$$

When $c=3, d=4$, calculating both sides by *Mathematica* for $p=3.5$,

```
Γ[p_] := Gamma[p]
Gl[c_, d_, p_] := 1/Γ[p] ∫_2^10 (10 - x)^(p-1) Sech[c x + d] dx
Gr[c_, d_, p_] := (1/c)^p 1/Γ[p] ∫_(2 c+d)^(10 c+d) (10 c + d - x)^(p-1) Sech[x] dx
N[{Gl[3, 4, 3.5], Gr[3, 4, 3.5]}]
{0.00148737      , 0.00148737}
```

It turns out that both sides are equal.

Setting $a=\pm\infty$, $b=0$ in Theorem 30.3.1 , we obtain the following corollary.

Corollary 30.3.2

Let c, p are a positive numbers and d be a real numbers, If a super primitive function $g^{<p>}(x)$ of a function $g(x)$ is bounded and holomorphic at an interval $(\pm\infty, d]$, the following holds.

$$\frac{1}{\Gamma(p)}\int_{\pm\infty}^{0} (0-x)^{p-1} g(cx+d)\,dx = \left(\frac{1}{c}\right)^p \frac{1}{\Gamma(p)}\int_{\pm\infty}^{d} (d-x)^{p-1} g(x)\,dx$$

Where, for $\pm$, the one where the integrals converge is used.

Example 2 $g(cx+d) = e^{cx+d}(cx+d)^{\lambda}$

According to the corollary, the p th order definite integral from $-\infty$ to 0 is as follows

$$\frac{1}{\Gamma(p)}\int_{-\infty}^{0} (-x)^{p-1} e^{ct+d}(cx+d)^{\lambda} dx = \left(\frac{1}{c}\right)^p \frac{1}{\Gamma(p)}\int_{-\infty}^{d} (d-x)^{p-1} e^x x^{\lambda} dx \quad (\lambda > -1)$$

When $c=3, d=4, \lambda=-0.99$, calculating both sides by *Mathematica* for $p=2.5$,

$$\Gamma[p\_] := \text{Gamma}[p]$$

$$\text{Gl}[c\_, d\_, \lambda\_, p\_] := \frac{1}{\Gamma[p]} \int_{-\infty}^{0} (0 - x)^{p-1} e^{c x+d} (c x + d)^{\lambda} \, dx$$

$$\text{Gr}[c\_, d\_, \lambda\_, p\_] := \left(\frac{1}{c}\right)^{p} \frac{1}{\Gamma[p]} \int_{-\infty}^{d} (d - x)^{p-1} e^{x} x^{\lambda} \, dx$$

N[{Gl[3, 4, -0.99, 2.5], Gr[3, 4, -0.99, 2.5]}]

{1.52252 - 1.21054 i, 1.52252 - 1.21054 i}

It turns out that both sides are equal.

2024.07.05

2024.07.08 Updated

Kano Kono

Hiroshima, Japan

Alien's Mathematics